Azonosító jel:

ÉRETTSÉGI VIZSGA ● 2016. május 3.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2016. május 3. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 240 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

8. Always state the final result (the answer to the question of the problem) in words, too!

9. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

10. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

11. Please, **do not write in the grey rectangles**.

I.

1. Solve the following equations in the set of real numbers.

a) $\frac{2x+11}{3}=\sqrt{x^2+6x+9}$

b) $\log_2(x+1)+\log_2(x-3)-\log_2(x+9)=1$

a)	6 points	
b)	7 points	
T.:	13 points	

2. Every student in a class of 28 received an end-of-year grade in both mathematics and physics. 23 students did not receive an “excellent” physics grade and 21 students did not receive an “excellent” mathematics grade. However, 10 students received an “excellent” grade in at least one of the two subjects.

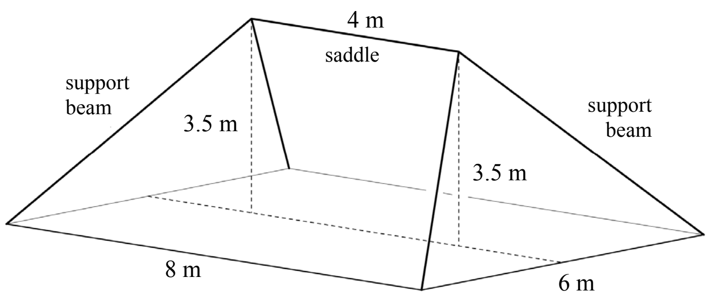
a) How many students got an “excellent” grade in both subjects?

There are two sets A and B. The cardinalities (i.e. the number of elements) of sets $A \setminus B$, $A \cap B$, A and B, in this particular order, form four consecutive terms of an increasing arithmetic sequence. The sum of the cardinalities of sets A and B is 28.

b) Give the first term and the common difference of this arithmetic sequence.

a)	4 points	
b)	7 points	
T.:	11 points	

3. A roof is being constructed atop a rectangular building. The length of the building is 8 metres and the width is 6 metres. The 4 metre long saddle sits right above the longer centreline of the rectangle, at a height of 3.5 metres above it. It is supported by four beams of equal length, sprung from each vertex of the rectangle.



a) Calculate the length of the support beams and the angle they make with the horizontal.

One trapezoidal face of the roof points towards South. A rectangular solar panel is attached to this face such that one side of the panel is on the bottom edge of the roof, while the opposite side is on the midsegment of the trapezium. The entire solar panel is within the trapezium of the roof's face.

b) What is the area of the largest solar panel that could be fitted onto the roof as described above?
Give your answer in square metres, rounded to one decimal place.

a)	7 points	
b)	6 points	
T.:	13 points	

4. The management of the handball team of a city would like to increase the income from ticket sales. Data accumulated over the years shows that an average 1000 tickets are sold per game if the ticket price is 1500 Ft. Analysis of the data also reveals that every 5-forint decrease in ticket price sells another 10 tickets, on average, while every 5-forint increase in ticket price results in another 10 tickets less sold. (The forint amount of the ticket price ends in a 0 or 5.)

a) Prove that, according to the above model and given that the present ticket price is 1500 Ft, any increase in this price will necessarily decrease the total income from ticket sales.

b) Using the above model, give the maximum of the total income from ticket sales and the ticket price that belongs to this maximum total income.

a)	6 points	
b)	8 points	
T.:	14 points	

II.

You are required to solve any four of the problems 5 to 9. Enter the number of the problem not selected in the blank square on page 3.

5. A factory produces identical shirts on two automated machines. 2% of the 4000 shirts made on the first machine are defective, while on the second machine 3.4% of 5000 shirts are defective. All shirts were delivered into the same warehouse where they were all mixed up. One of the 9000 shirts was randomly selected and found defective.

a) What is the probability that this defective shirt was made on the second machine?

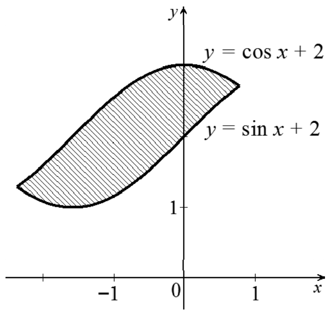
A department store first offered a 500-forint discount for a defective shirt and then, later, this has been further decreased by another p% of the discount price. The final price is now 50 Ft more than it would be, should they have first decreased the price by p% and then give the 500-forint discount. On the other hand, this final price is still 90 Ft less than it would be after decreasing the original price by p% twice.

b) Calculate the value of p and the original price of the shirt.

a)	5 points	
b)	11 points	
T.:	16 points	

You are required to solve any four of the problems 5 to 9. Enter the number of the problem not selected in the blank square on page 3.

6. **a)** Calculate the area of the shaded region between the two graphs shown in the diagram.
(One of the graphs is part of the graph of the function $y = \sin x + 2$, the other is part of $y = \cos x + 2$.)



b) Prove that the sequence $a_n = \frac{11n-5}{3n-8}$ is not monotonic but bounded. ($n \in \mathbf{N}^+$)

a)	8 points	
b)	8 points	
T.:	16 points	

You are required to solve any four of the problems 5 to 9. Enter the number of the problem not selected in the blank square on page 3.

7. **a)** Determine the number of positive integers that are less than 1000, none of their digits is 0, but at least one of their digits is 1.

A set of data contains positive integers. The mode of the data is 32, the mean is 22, the smallest number among the data is 10. Median m is one of the data, its frequency is 1. Replace m by $(m + 10)$, and the mean changes to 24. Replace m by $(m - 5)$, and the median would change to $m - 4$.

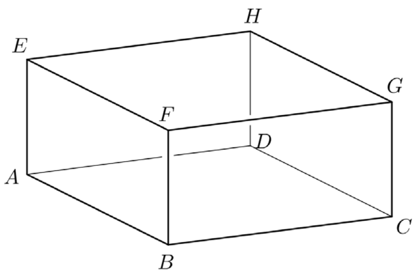
b) Prove that there are five numbers in this set of data.

c) Determine the elements of the original set of data.

a)	6 points	
b)	2 points	
c)	8 points	
T.:	16 points	

You are required to solve any four of the problems 5 to 9. Enter the number of the problem not selected in the blank square on page 3.

8. In cuboid *ABCDEFGH* the edges perpendicular to the face *ABCD* are *AE*, *BF*, *CG* and *DH*. The lengths of the edges of the cuboid are $AB = 12$ cm, $AD = 16$ cm, and $AE = 5$ cm.



a) Calculate the volume of tetrahedron *ACFH*.

b) Prove that the faces of tetrahedron *ACFH* are congruent triangles.

c) Prove that the faces of tetrahedron *ACFH* are acute triangles.

The length of edge *QP* of the tetrahedron *PQRS* is 10 cm, $PS = 15$ cm, $SR = 40$ cm. The lengths of the other three edges are 20 cm, 25 cm, and 30 cm.

d) How many different tetrahedrons meet the above criteria? (Congruent tetrahedrons are not considered different.)

a)	4 points	
b)	3 points	
c)	5 points	
d)	4 points	
T.:	16 points	

You are required to solve any four of the problems 5 to 9. Enter the number of the problem not selected in the blank square on page 3.

9. A token is moved along a straight path in a board game. Starting from the field marked "Start" the token is moved forward 1, 2, 3, 4, 5, or 6 fields according to the number rolled on a fair dice. If the token lands on field 4 at any time during the game it must return to the Start field and restart the game from there. In this game, the only way of moving the token backwards is by landing on the field 4.

Start	1	2	3	**4**	5	6	7	…

a) What is the probability of landing on field 4 at least once during the game?

András has already moved his token three times and, just before his fourth move, he is now back on the Start field.

b) In how many different ways could András have moved his token during his first three moves?

a)	9 points	
b)	7 points	
T.:	16 points	

	Number of problem	maximum score	points awarded	maximum score	points awarded
Part I	1.	13		**51**	
	2.	11			
	3.	13			
	4.	14			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

	elért pontszám **egész számra** kerekítve/ score rounded to the nearest **integer**	programba beírt **egész** pontszám/ **integer** score entered into the program
I. rész/Part I		
II. rész/Part II		

javító tanár/examiner

jegyző/registrar

dátum/date

dátum/date